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An imperfect repair model based on reduction of virtual age and uniform distributed repair degrees

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Introduction

Repair strategies

- Perfect repair (as good as new) : (PR)
- Minimal repair (as bad as old) : (MR)
- Imperfect repair : (IR)

Introduction

- Nakagawa (1979-1980) - Optimum Preventive Maintenance policies.
  The imperfect PM is modeled in this way: after PM a unit is returned to the "as good as new" state (perfect PM) with probability p and returned to the "as bad as old" state (minimal PM) with probability $(1 - p)$.
- Brown and Proschan (1983) - Repair is perfect with probability p and minimal with probability $(1 - p)$.
- Kahle(1991) - Simultaneous confidence region (case of perfect repair and case of minimal repair).
- Bathe and Franz (1996) - Modelling of repairable systems with various degrees of repair.

Introduction

- Last and Szekli (1998) - Stochastic comparison of repairable systems by coupling.
- Kijima (1988,1989) - Virtual age models or Generalized Renewal Process (GRP).
- Baxter, Kijima, and Tortorella (1996) - Generalization of Kijima's models.
- Gasmi, Love and Kahle (2003) - Modelling and estimation of repair effects of complex repairable systems.
- Doyen and Gaudoin (2004) - ARA and ARI models based on an Arithmetic Reduction of virtual Age or failure Intensity.

Aim

- Aim: Establish general statistical models for repairable systems
- Concept: virtual age
  The first who discovered this process are *Malik* (1979), *Kijima, Morimura and Suzuki* (1988), *Kijima* (1989) and *Stadje & Zuckerman* (1991).
- Advantages
  - flexibility for modeling repairable systems.
  - better representation of statistical models of the real situation.

Basic models

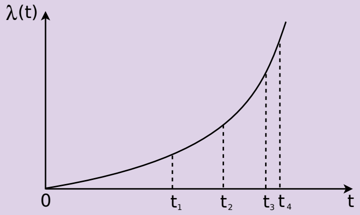
Basic models

1. Minimal maintenance model
2. Perfect maintenance model

(1) Minimal maintenance model

- the maintenance effect is to put the system in operation in the same state just before failure.
- the system is said (ABAO).
- the failure intensity $\lambda(t)$ does not depend on the past of the process.

(1) Failure intensity in the case MR



Basic models

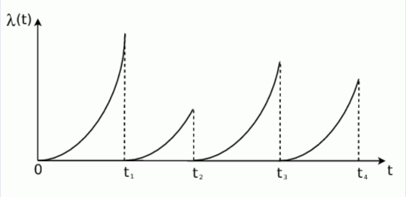
Basic models

1. Minimal maintenance model
2. Perfect maintenance model

(2) Perfect maintenance model

- the maintenance effect is to put the system into operation in the same state as new.
- the system is said (AGAN).
- operating times are independent and identically distributed.
- the failure intensity $\lambda(t)$ depends on the past of the process.

(2) Failure intensity in the case PR



Imperfect maintenance model

Imperfect maintenance model

- In practice, the situation is between the two extreme cases:
  - minimal maintenance (ABAO)
  - perfect maintenance (AGAN)
- Industrial systems are difficult to refurbish after maintenance
- In the industrial field the maintenance has an effect more than minimal

Remark

We can note that most of the models concerning the modeling of repairable systems identify the minimal repair and the imperfect repair actions. Naturally, this popular assumption is a very unreal one.

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The Model

Description of the general model

$\lambda_1(t) = \lambda(t)$	for $t \in [0, t_1)$.
$\triangleright$ At t_1 failure	$\rightarrow$ random degree of repair $z_1 \in [0,1]$
Virtual age in t_1	$\rightarrow$ $v_1 = z_1 t_1$.
$\lambda_2(t) = \lambda(t - t_1 + v_1)$	with $t \in [t_1, t_2)$.
$\triangleright$ At t_2 failure	$\rightarrow$ random degree of repair $z_2 \in [0,1]$
Virtual age in t_2	$\rightarrow$ $v_2 = z_2(v_1 + t_2 - t_1)$.

$$\vdots$$

The general model

Description of the general model

$$\lambda_{k+1}(t) = \lambda(t - t_k + v_k) \qquad \text{for } t_k \leq t < t_{k+1}$$

$$v_k = z_k(v_{k-1} + t_k - t_{k-1}), \quad v_0 := 0\,;\; t_0 := 0\,;$$

Definition

$v(t) := t - t_k + v_k$, $t_k \leq t < t_{k+1}$, $k \geq 1$, is called the virtual age Process.

Virtual Age Process

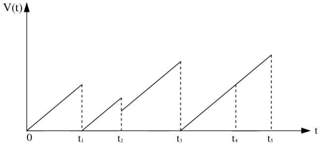

Figure 1: Virtual Age Process

Failure Intensity

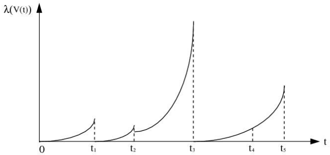

Figure 2: Failure Intensity

Special cases Model

Special cases Model

- Perfect maintenance model
- Minimal maintenance model
- Kijima's models (1989)
- Brown and Proschan model (1983)
- Stadje and Zuckerman model (1991)
- Reliability model with alternating repairs (Gasmi (2011))

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Assumptions of the model

1. Repairs affect the failure intensity at any instant via a virtual age process from type Kijima 2.
2. After failure, one of the three following cases is possible:
   - perfect repair
   - minimal repair
   - imperfect repair with uniform distributed degree of repair
3. All repair times are small and can be neglected.
4. The baseline failure intensity of the system is from Weibull type:

$$\lambda(x, \theta) \quad = \quad \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} \quad , \alpha > 0 \, , \; \beta > 0 \, ,$$

with $\theta = (\alpha, \beta)$, α scale parameter, β form parameter.

The imperfect repair model based on uniform distributed repair degrees

- $(t_k)_{k=1,2,\ldots}$ failure times
- $(z_k)_{k=1,2,\ldots}$ repair degrees
- $N(t) = \sum_{k=1}^{\infty} \mathbf{1}(t_k \leq t)$ the number of failures until t for the failure repair process, with $\mathbf{1}(A)$ is the indicator function of A

◇ If the degree of repairs $z_k \in (0,1)$, $\forall k = 1, \ldots, N(t)$ we obtain imperfect repairs. (model 1)

◇ If the degree of repairs $z_k = 0$, $\forall k = 1, \ldots, N(t)$ we obtain only perfect repairs. (model 2)

◇ If the degree of repairs $z_k = 1$, $\forall k = 1, \ldots, N(t)$ we obtain only minimal repairs. (model 3)

The imperfect repair model based on uniform distributed repair degrees

Description

We consider a marked point process $\Phi = ((t_k, z_k))$. Φ is described by:

1. the counting process $\{N(t),\ t \geq 0\}$ and the corresponding intensities $\lambda(v(t), \theta)$
   with $V(t) := t - t_k + v_k, \quad t_k \leq t < t_{k+1}\ ,\ k = 1, 2, \ldots$ is the virtual age process.
2. the marks z_k are repair degrees at t_k.

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The LL function

The log-likelihood function for observation of point processes is of the form (Liptser and Shiryayev (1978))

$$\ln L(t;\theta) \quad = \quad \sum_{k=1}^{N(t)} \ln \lambda(v(t_k-)) - \int_0^t \lambda(v(s))ds,$$

where $v(t_k-) = t_k - t_{k-1} + v_{k-1}$.

Parameter Estimation

The LL function

We obtain:

$$\ln L(t;\theta) \;=\; (\beta-1)\sum_{i=1}^{N(t)} \ln(v_{k-1}+t_k-t_{k-1}) + N(t)(\ln\beta-\beta\ln\alpha)-\frac{1}{\alpha^{\beta}}S_1.$$

Where

$$S_1=\sum_{k=1}^{N(t)}\left\{(v_{k-1}+t_k-t_{k-1})^{\beta}-(v_k)^{\beta}\right\}+(t-t_{N(t)}+v_{N(t)})^{\beta}.$$

Parameter Estimation

The MLE of the parameters α and β are obtained by solving the nonlinear system:

$$\frac{\partial \ln L(t;\theta)}{\partial \alpha} = 0$$

and

$$\frac{\partial \ln L(t;\theta)}{\partial \beta} = 0.$$

Parameter Estimation

- The estimator of the scale parameter α is explicitly determined:

$$\hat{\alpha} = \left(\frac{S_1|_{\hat{\beta}}}{N(t)} \right)^{1/\hat{\beta}}$$

- $\hat{\alpha}$ involves the usual parameter estimation in terms of Weibull intensities.
- This estimator depends on the virtual age of the system and the number of failures $N(t)$.

Parameter Estimation

- The estimator of the shape parameter β can be found by numerical solve of the following equation:

$$\frac{1}{\hat{\beta}} + \frac{1}{N(t)} \left\{ \sum_{k=1}^{N(t)} \ln(v_{k-1} + t_k - t_{k-1}) \right\} - \frac{S_2|_{\hat{\beta}}}{S_1|_{\hat{\beta}}} = 0 \,.$$

Where

$$S_2 = \sum_{k=1}^{N(t)} \left\{ (v_{k-1} + t_k - t_{k-1})^{\beta} \ln(v_{k-1} + t_k - t_{k-1}) - v_k^{\beta} \ln v_k \right\}$$

$$+(t - t_{N(t)} + v_{N(t)})^{\beta} \ln(t - t_{N(t)} + V_{N(t)}).$$

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Estimation of the Fisher Information

- Because the MLE of the vector $\theta = (\alpha, \beta)$ is not obtained in closed form, it is not possible to derive the exact distribution of the MLE.
- An approximation of the Fisher information matrix $I(\theta)$ is given.
- In this case n independent failure repair processes are observed.
- Let $\theta = (\theta_1, \theta_2)$ with $\theta_1 = \alpha$ and $\theta_2 = \beta$.
- The elements of the 2x2 matrix $I(\theta)$, $I_{r,s}(\theta)$, $r, s = 1, 2$, can be approximated by $\widehat{I_{r,s}(\theta)} = -\frac{1}{n}\sum_{l=1}^{n} \frac{\partial^2 \ln L_l(t;\theta)}{\partial\theta_r \partial\theta_s}$.

Estimation of the Fisher Information

- Let $l \in \{1, 2, \ldots, n\}$.

Notations:

- $L_l(t;\theta)$ – the likelihood function of the l-th failure repair process.
- $N^l(t)$ – the number of failures until t for the l-th failure repair process.
- $t_{l,1}, \ldots, t_{l,N^l(t)}$ – failure times of the l-th failure repair process.
- $x_{l,1}, \ldots, x_{l,N^l(t)}$ – operating times of the l-th failure repair process.
- $v_{l,1}, \ldots, v_{l,N^l(t)}$ – virtual age of the l-th failure repair process.

Definition

Let $i = 1, 2, \ldots, N^l(t)$, $x_{l,i} = t_{l,i} - t_{l,i-1}$ is the operating time between two successive failures of the l-th failure repair process.

Estimation of the Fisher Information

- The observed information matrix I for this model is given by:

$$I = \begin{pmatrix} -\frac{1}{n}\sum_{l=1}^{n}\frac{\partial^2 \ln L_l(x,\theta)}{\partial\alpha^2} & -\frac{1}{n}\sum_{l=1}^{n}\frac{\partial^2 \ln L_l(x,\theta)}{\partial\alpha\partial\beta} \\ -\frac{1}{n}\sum_{l=1}^{n}\frac{\partial^2 \ln L_l(x,\theta)}{\partial\beta\partial\alpha} & -\frac{1}{n}\sum_{l=1}^{n}\frac{\partial^2 \ln L_l(x,\theta)}{\partial\beta^2} \end{pmatrix}$$

- The variance-covariance matrix V is the inversion of the observed information matrix I

$$V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} = I^{-1}$$

Estimation of the Fisher Information

• It follows then that the asymptotic distribution of the MLE $(\hat{\alpha}, \hat{\beta})$ (Miller (1981)):

$$\begin{pmatrix} \hat{\alpha} \\ \hat{\beta} \end{pmatrix} \sim N\left(\begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}\right)$$

• If we replace the parameters α, β by the corresponding MLE's, we get then an estimate of the variance-covariance matrix V, denoted by $\hat{V}$ and defined as follows:

$$\hat{V} = \begin{pmatrix} \widehat{I_{11}} & \widehat{I_{12}} \\ \widehat{I_{21}} & \widehat{I_{22}} \end{pmatrix}^{-1}$$

with $\widehat{I_{ij}} = I_{ij}$ are obtained if we replace (α, β) by $(\hat{\alpha}, \hat{\beta})$.

Estimation of the Fisher Information

- We obtain then approximate $100(1-\nu)\%$ confidence intervals for the parameters α, β respectively as:

$$\hat{\alpha} \pm z_{\frac{\nu}{2}} \sqrt{\widehat{V_{11}}}, \quad \hat{\beta} \pm z_{\frac{\nu}{2}} \sqrt{\widehat{V_{22}}},$$

where $z_{\frac{\nu}{2}}$ is the upper $\frac{\nu}{2}$-th percentile of the standard normal distribution.

Estimation of the Fisher Information

Theorem

$$\widehat{I_{1,1}(\theta)} = \frac{\hat{\beta}^2}{n\hat{\alpha}^2} \sum_{l=1}^{n} N^l(t),$$

$$\widehat{I_{1,2}(\theta)} = -\frac{1}{n\hat{\alpha}} \sum_{l=1}^{n} \left\{ N^l(t)(1 - \hat{\beta}\ln\hat{\alpha}) + \hat{\beta} \sum_{i=1}^{N^l(t)} \ln(x_{l,i} + v_{l,i-1}) \right\},$$

$$\widehat{I_{2,2}(\theta)} = \frac{1}{n} \sum_{l=1}^{n} \left\{ N^l(t)\left(\frac{1}{\hat{\beta}} - \ln\hat{\alpha}\right)^2 - 2\ln\hat{\alpha} \sum_{i=1}^{N^l(t)} \ln(x_{l,i} + v_{l,i-1}) + \frac{1}{\hat{\alpha}^{\hat{\beta}}} W_{3,l}(t,\hat{\beta}) \right\}.$$

Estimation of the Fisher Information

Where

$$W_{3,l}(t,\hat{\beta}) = \sum_{i=1}^{N^l(t)} \left\{ (x_{l,i} + v_{l,i-1})^{\hat{\beta}} \ln^2(x_{l,i} + v_{l,i-1}) - (v_{l,i})^{\hat{\beta}} \ln^2(v_{l,i}) \right\}$$
$$+ (t - t_{l,N^l(t)} + v_{l,N^l(t)})^{\hat{\beta}} \ln^2(t - t_{l,N^l(t)} + v_{l,N^l(t)}).$$

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Simultaneous confidence region based on the likelihood ratio

LR

- Method: Likelihood ratio (LR)
- Under regularity conditions (Barndhoff-Nielsen and Blaesild, (1986)) the log-likelihood ratio (LLR) $q = 2\left\{\ln L(x, \hat{\theta}) - \ln L(x, \theta)\right\}$ converges in distribution to a central χ^2 distribution with 2 degrees of freedom.
- The simultaneous confidence region is defined by the inequality $q \leq \chi^2_{1-\mu,2}$.
- $\chi^2_{1-\mu,2} = -2\ln\mu$ is the $(1-\mu)$- quantile of the χ^2- distribution with 2 degrees of freedom.

Simultaneous confidence region based on the likelihood ratio

- We study the case of n independant failure repair process.
- The border of the simultaneous confidence region using the likelihood ratio is given as follows:

$$2\{\ln L_r(t;\hat{\theta}) - \ln L_r(t;\theta)\} = -2\ln\mu.$$

- Notations:
  - $N^r(t)$ - the number of failures until t for the r-th failure repair process.
  - $t_{r,1},\ldots,t_{r,N^r(t)}$ - failure times of the r-th failure repair process.
  - $v_{r,1},\ldots,v_{r,N^r(t)}$ - virtual ages of failures until t for the r-th failure repair process.

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  - $v_{r,1},\ldots,v_{r,N^r(t)}$ - virtual ages of failures until t for the r-th failure repair process.

Simultaneous confidence region based on the likelihood ratio

- We obtain then the following simultaneous confidence region for n independent failure repair processes:

$$\frac{1}{\alpha^{\beta}}\sum_{r=1}^{n} S_{r,1} + (\hat{\beta}-\beta)\sum_{r=1}^{n}\sum_{k=1}^{N^r(t)} \ln(v_{r,k-1}+t_{r,k}-t_{r,k-1})$$

$$+(\ln\hat{\beta}-\ln\beta-1-\hat{\beta}\ln\hat{\alpha}+\beta\ln\alpha)\sum_{r=1}^{n} N^r(t) = -\ln\mu.$$

where

$$S_{r,1} = \sum_{k=1}^{N^r(t)}\left\{(v_{r,k-1}+t_{r,k}-t_{r,k-1})^{\beta}-(v_{r,k})^{\beta}\right\}+(t-t_{N^r(t)}+v_{N^r(t)})^{\beta}.$$

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Simulation study

Example 1

- The simulation study to generate data of the IR process and following the Kijima type 2 repair model with uniform distributed degrees of repair.
- Let $\alpha = 1.2$ and $\beta = 3$.
- We obtain $N(t) = 10$ failures until the time $t = 10$.

t_k	1.3515	1.8118	2.4047	3.3637	3.4218
zk	0.4159	0.4900	0.3486	0.6980	0.1740
t_k	4.1991	4.6912	5.6913	7.9375	8.0719
zk	0.2247	0.8906	0.1236	0.1357	0.4553

Table 1: Failure times and repair degrees for $\alpha = 1.2$ and $\beta = 3$

Simulation study

- A sample 1 was observed until $t = 100$. Let $\alpha = 1.2$, $\beta = 3$ and $s = 10$, where s is the number of simulations.

$\hat{\alpha}$	$\hat{\beta}$	LL
1.1907	2.9978	-25.0579
1.2014	2.9880	-24.9123
1.2337	3.0314	-26.5588
1.2216	3.1723	-26.7767
1.2537	3.0977	-28.6813
1.2158	2.9855	-26.9009
1.1789	2.9456	-26.6481
1.1666	2.9134	-28.7947
1.1855	3.0615	-25.9890
1.2104	2.9219	-26.1210

Table 2: Estimations of α, β and LL from data of sample 1

Simulation study

- The mean squared errors (MSE) of $\hat{\alpha}$ and $\hat{\beta}$ are given in Table 3.

s	50	100	500	1000
MSE($\hat{\alpha}$)	0.0022	0.0018	0.0017	0.0015
MSE($\hat{\beta}$)	0.0993	0.0542	0.0514	0.0504

Table 3: MSE of $\hat{\alpha}$ and $\hat{\beta}$

- We could remark that if the number of simulations s increases, then the mean squared errors of $\hat{\alpha}$ and $\hat{\beta}$ decrease.

Simulation study

• A sample 2 was observed until $t = 100$. Let $\alpha = 1.2$, $\beta = 3$ and $s = 200$, where s is the number of simulations.

• The bias and variance of the estimators are estimated by their empirical version on 200 replicates.

• The estimations are given in Table 4.

	$\hat{\alpha}$	$\hat{\beta}$
Estimation	1.1903	2.9896
Empirical mean	1.2001	3.0370
Empirical variance	0.0016	0.0541

Table 4: Estimations results considering an average of 200 simulations

Simulation study

- A sample 1 was observed until $t = 100$. Let $\alpha = 1.2$, $\beta = 3$.
- The parameter estimates are $\hat{\alpha} = 1.1907$ and $\hat{\beta} = 2.9978$ and the $LL = -25.0579$.
- Figure 3 illustrates the LL function with respect to α and β.

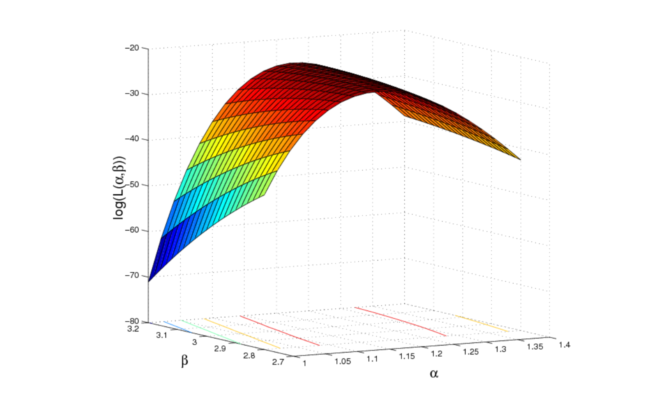

Figure 3: Graph of the LL function for one sample with respect to α and β

Simulation study

- Figure 4 illustrates simultaneous confidence region of the parameter estimations $\hat{\alpha} = 1.2030$ and $\hat{\beta} = 3.0375$ by given $\mu = 0.05$.

- for $n = 50$ (curve in dash).

- for $n = 100$ (curve in line).

Simulation study

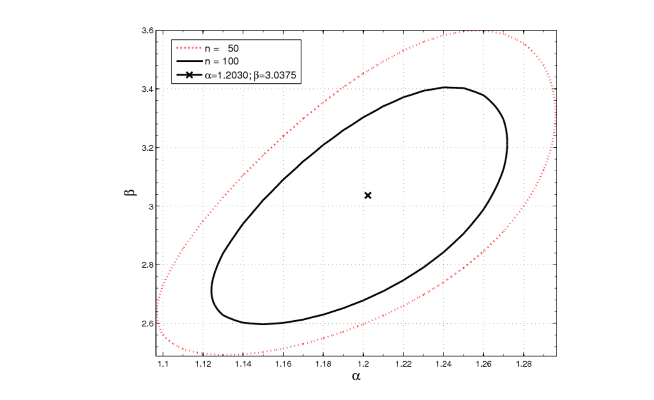

Figure 4: Simultaneous confidence region for the estimations of $\alpha = 1.2$ and $\beta = 3$

Simulation study

- The simultaneous confidence region based on the likelihood ratio for $n = 100$ is smaller than that for $n = 50$.

- In the case of $n = 100$, the parameter α varies between 1.124 and 1.270 and the parameter β varies between 2.6 and 3.4.

- In the case of $n = 50$, the parameter α varies between 1.096 and 1.296 and the parameter β varies between 2.5 and 3.6.

Simulation study

Example 2

- Figure 5 illustrates simultaneous confidence region of the parameter estimations $\hat{\alpha} = 1.4955$ and $\hat{\beta} = 3.4975$ by given $\mu = 0.05$.

- for $n = 50$ (curve in dash).

- for $n = 100$ (curve in line).

Example 2

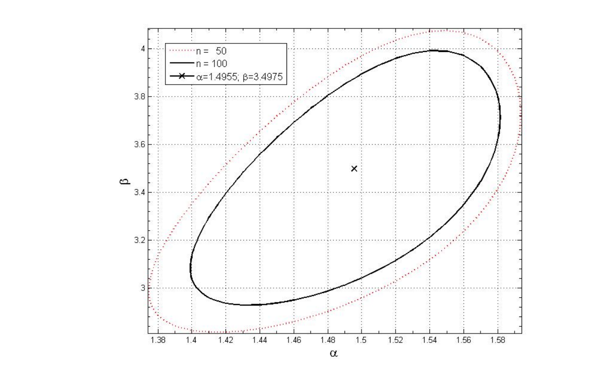

Figure 5: Simultaneous confidence region for the estimations of $\alpha = 1.5$ and $\beta = 3.5$

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Illustrative Example

- We provide a data analysis to investigate how this model works in practice.

- We illustrate the modeling and estimation procedure.

- A well known data on airplane air-conditioning failures on a fleet of Boeing aircraft (Plane 7914) given in Hollander and Wolfe are considered.

- For the data from Plane 7914, the number of failures is N(t)=24.

- The rest time after the last failure is assumed to be equal to zero.

Illustrative Example

- Our objective is to compare the model with uniform distributed degree of repair, denoted by model 1 and models using a fixed degree of repair.

- Models using a fixed degree of repair:
  1. The model 2 (perfect repairs); RP.
  2. The model 3 (minimal repairs); NHPP.
  3. The model 4 (average repairs); (degrees of repair $z_k = 0.5$).

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Illustrative Example

- Table 5 gives the inter-failure times for Plane 7914 and the uniform distributed degrees of repair used in model 1.

i	1	2	3	4	5	6	7	8
x_i	50	44	102	72	22	39	3	15
z_k	0.83	0.63	0.54	0.65	0.73	0.09	0.87	0.01
i	9	10	11	12	13	14	15	16
xi	197	188	79	88	46	5	5	36
z_k	0.29	0.18	0.93	0.07	0.58	0.64	0.65	0.86
i	17	18	19	20	21	22	23	24
xi	22	139	210	97	30	23	13	14
z_k	0.05	0.81	0.53	0.69	0.21	0.54	0.70	0.96

Table 5: Inter-failure times from Plane 7914 and degrees of repair (model 1)

Illustrative Example

• For comparison purpose, we use the mean square of the difference between the empirical cdf and the fitted cdf, say MSD.

$$MSD = \frac{1}{N(t)} \sum_{k=1}^{N(t)} \left(\hat{F}_k - F_{E,k} \right)^2 ,$$

- $\hat{F}_k \cdots$ the empirical cdf computed at the cumulative failure times t_k.
- $F_{E,k} \cdots$ the estimated cdf computed at the cumulative failure times t_k.

Illustrative Example

- The ML estimates of the parameters α and β, the MSD and the LL values are given in Table 6.

	$\hat{\alpha}$	$\hat{\beta}$	MSD	LL
Model 1	56.8875	0.9224	0.0010	-123.8080
Model 2	65.4085	1.0339	0.0014	-123.9939
Model 3	82.9261	1.0880	0.0057	-123.9455
Model 4	55.8537	0.9138	0.0011	-123.7917

Table 6: Estimations of α and β, MSD and LL from data of Plane 7914

Simulation study

- For all introduced models, the empirical, the estimated cdf and the 95% lower and upper confidence bounds for the cdf of the data from Plane 7914 are shown in Figure 6.

Simulation study

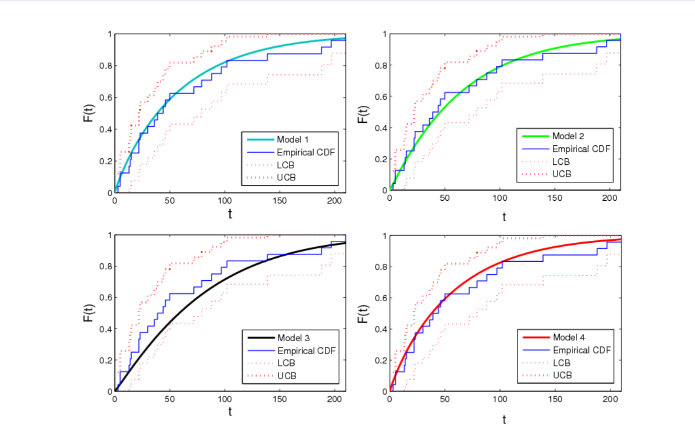

Figure 6: Cdf and empirical cdf of data from Plane 7914 for models 1-4

Simulation study

Based on Table 6 and Figure 6 we can conclude that:

1. Model 4 fits the data better than Model 2 and Model 3.
2. Model 1 fits the data better than Model 4.
3. Model 3 gives the worst fit of the data.
4. Model 1 concord the data better than all other models.

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Conclusions and Future Work

Conclusions

1. Parameter estimation of an imperfect repair model is proposed.
2. Simultaneous confidence region based on the likelihood ratio for the parameters of the Weibull intensity is developed in the case of uniform distributed degrees of repair.
3. The obtained results are applied on sets of simulated data.
4. Interesting results on real data are obtained.

Conclusions and Future Work

Future Work

1. Study other models.
2. Change the distribution of repair degrees.
   - ◇ Beta distribution with equal shape parameters.
   - ◇ Beta distribution by fixing one of its parameters.
   - ◇ Beta distribution.
3. Include time dependent repair effectiveness.
4. Study the Kijima's type 1 virtual age imperfect repair model and compare results.
5. Compare models with AIC criterion.

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Thank you for your attention