Multi-branch hidden state models for multi-mode deterioration modelling and RUL estimation

Thanh Trung LE, Christophe Bérenguer, Florent Chatelain

Team SAIGA, GIPSA-lab, Grenoble INP

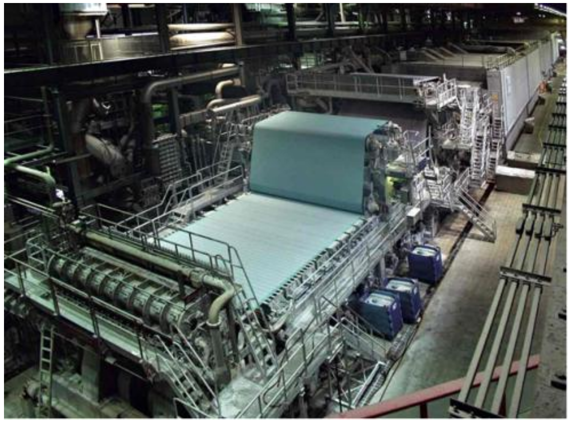
Wednesday, January 20, 2016

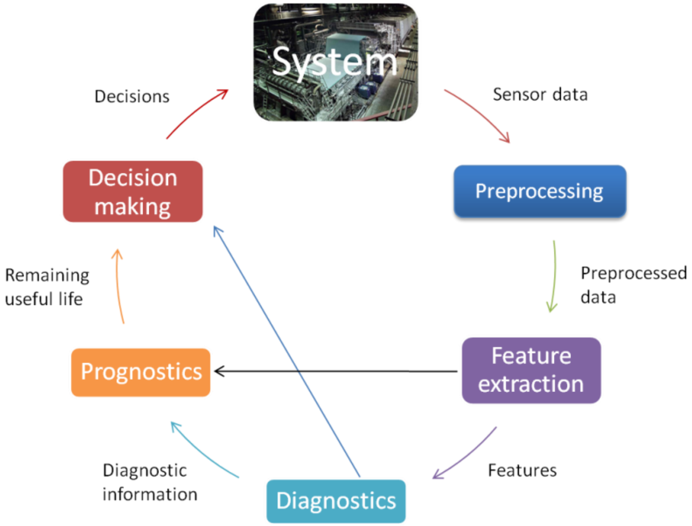
Industrial context

Continuous production systems

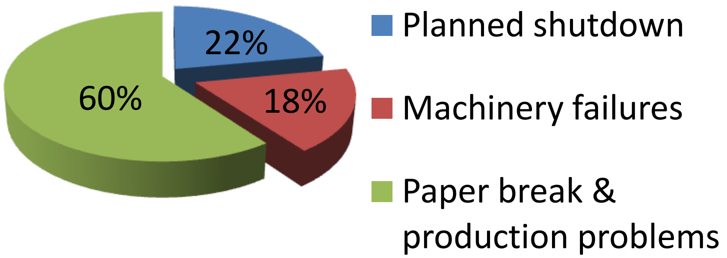
- Complex
- Continuity is a critical issue
- Can fail because of a defective component
- Components deteriorate over time

⇒ **Predictive maintenance**





Paper machine down time



Lost ~ **34 M€/factory/year**

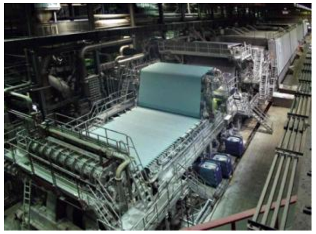
SUPREME project

SUPREME

FP7 european project

- “SUstainable PREdictive Maintenance for manufacturing Equipment”
- 09/2012 -> 08/2015
- 10 partners from both industry and academy
- **Purpose**: Development of new tools for predictive maintenance to improve productivity, reduce machine downtimes and increase energy efficiency.

⇒ Application case: Paper machine



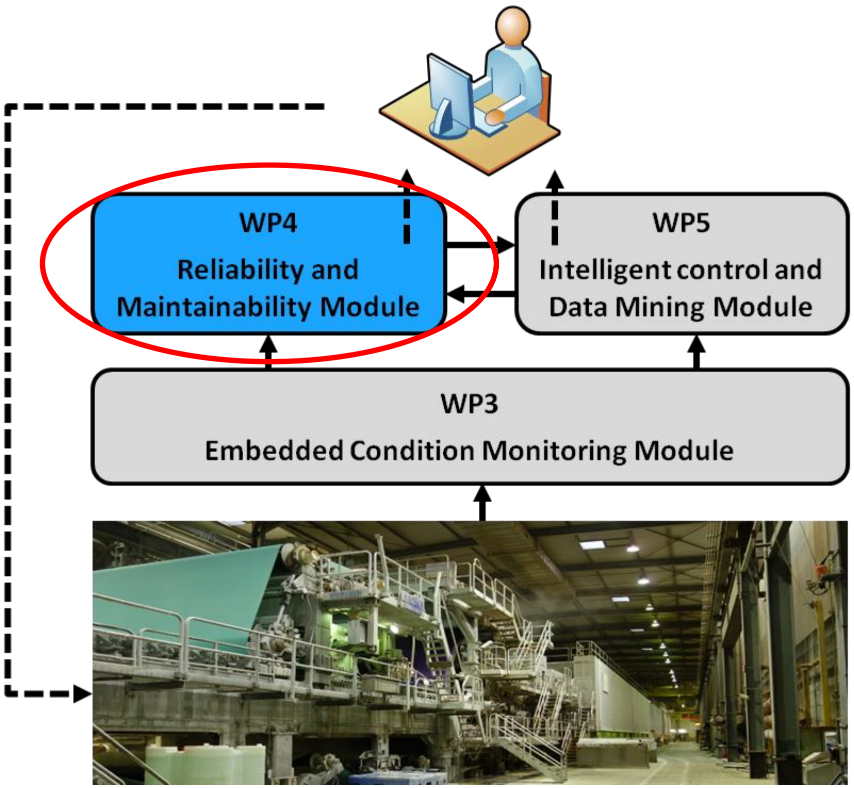
cetim COFELY ENDEL GDF SUEZ EC SYSTEMS Fraunhofer IPA Grenoble INP LECTA Loy & Hutz creating future OPTIMITIVE ORLOGA INGENIERIA CZECH TECHNICAL UNIVERSITY IN PRAGUE

This project has received funding from the European Community's Seventh Framework Programme (FP7-2012-NMP-ICT-FoF) under grant agreement no 314311

SUPREME partners

Reliability and maintainability module

- Systematic critical component identification based on risk assessment
- Deterioration-based reliability
- Dynamic adaption of maintenance strategy



SUPREME work packages

Objectives of works

- ✓ **Deterioration modeling**
- ✓ **Remaining Useful Life (RUL) estimation**

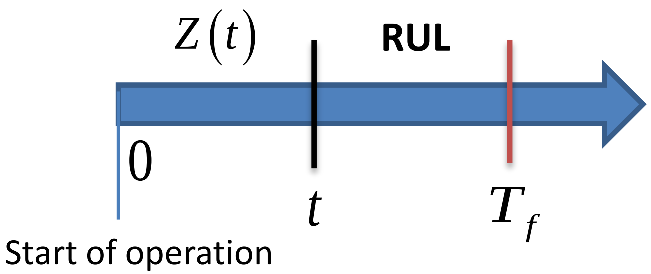
Outline

- Problem statement
- Multi-branch discrete-state models
- Jump Markov linear systems
- Conclusion & Perspectives

Outline

- Problem statement
- Multi-branch discrete-state models
- Jump Markov linear systems
- Conclusion & Perspectives

Remaining Useful Life (RUL)



- Residual time before the failure
- Conditional random variable:

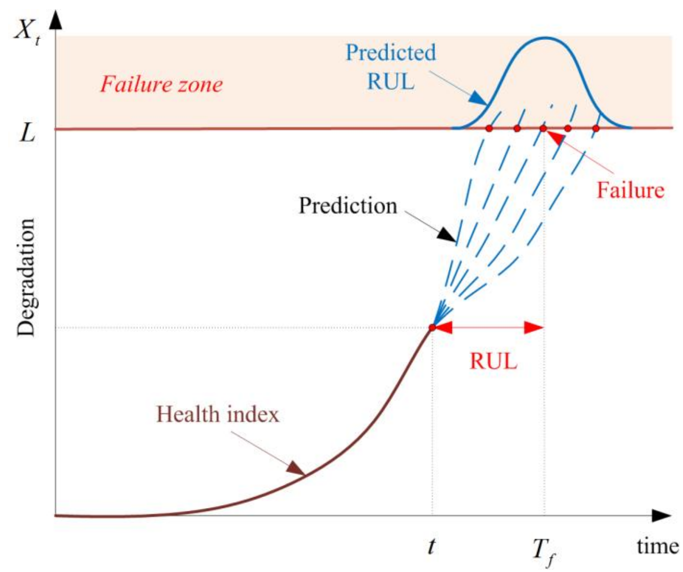
$$RUL = T_f - t \mid T_f > t, Z(t)$$

Z(t) : information up to time *t* ;

T_f : time to failure

- Uncertainties assessment: probabilistic distribution
- RUL estimation: propagation of health indicator toward threshold

=> Need of **deterioration models**

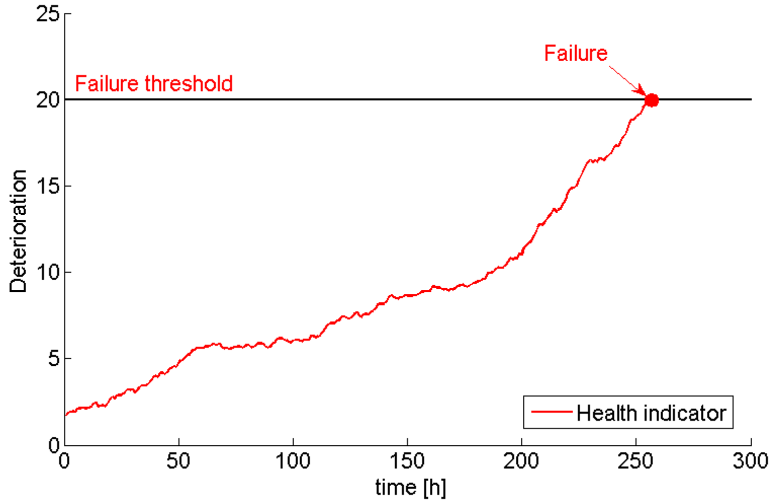


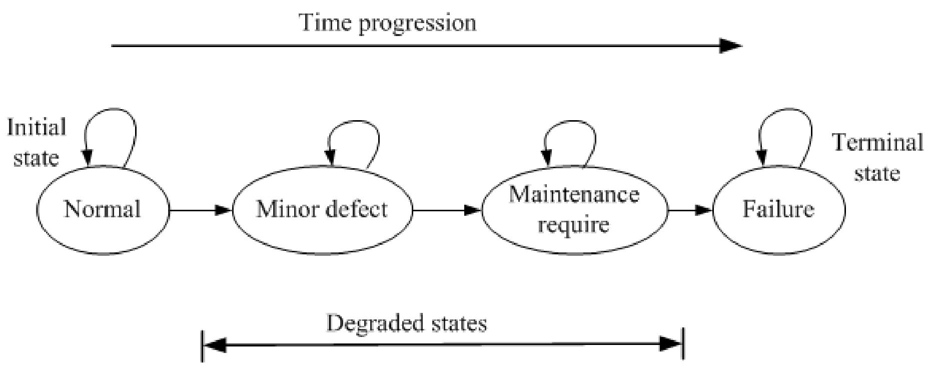
RUL estimation illustration

Deterioration modeling

- Model the dynamic stochastic behavior of the deterioration
- Link the failure of an item to its deterioration level
- Allow health state assessment
- Basis for RUL prediction

Classification





Continuous-state models

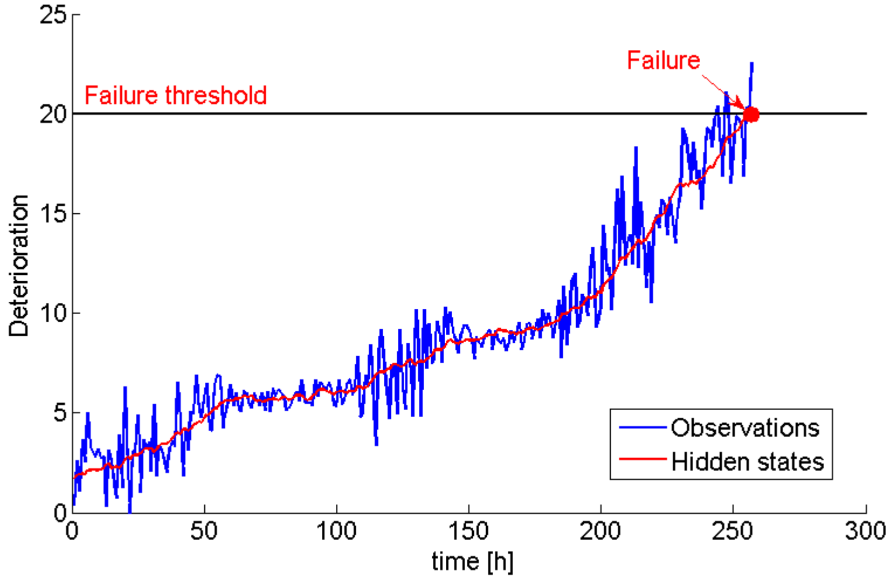
Discrete-state models

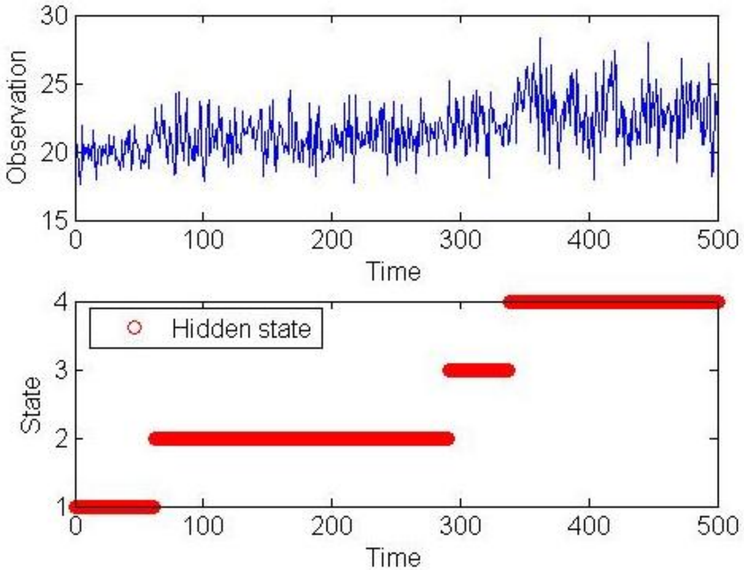
Problems

- Health states are difficult to measure or observe directly
- Observations are available, i.e. extracted features from raw condition monitoring data

=> State-space representation

$$\begin{cases} x_t = f(x_{t-1}, \omega_t) & : \text{Hidden states} \\ y_t = g(x_t, v_t) & : \text{Observations} \end{cases}$$





Continuous states: stochastic filtering

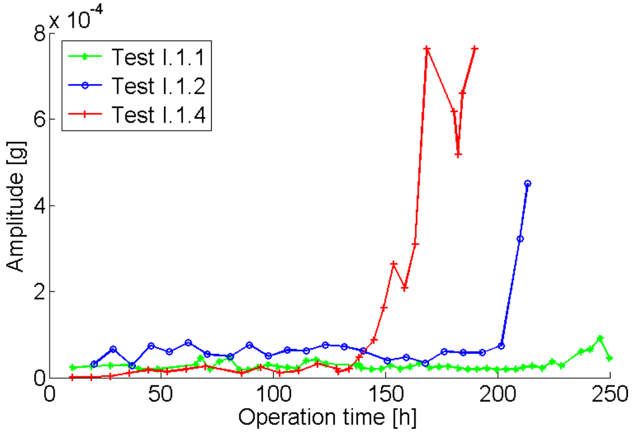
Discrete states: hidden markov model

Problems

- Co-existence of multiple modes, even within a single component

  Examples: deterioration rates depend on initiation time, on applied load, etc.

Test bench at CETIM



Bearing health index evolution

⟹ Proposed solution: Multi-branch modeling

- Discrete-state: Multi-branch Hidden Markov models
- Continuous-state: Jump Markov linear systems

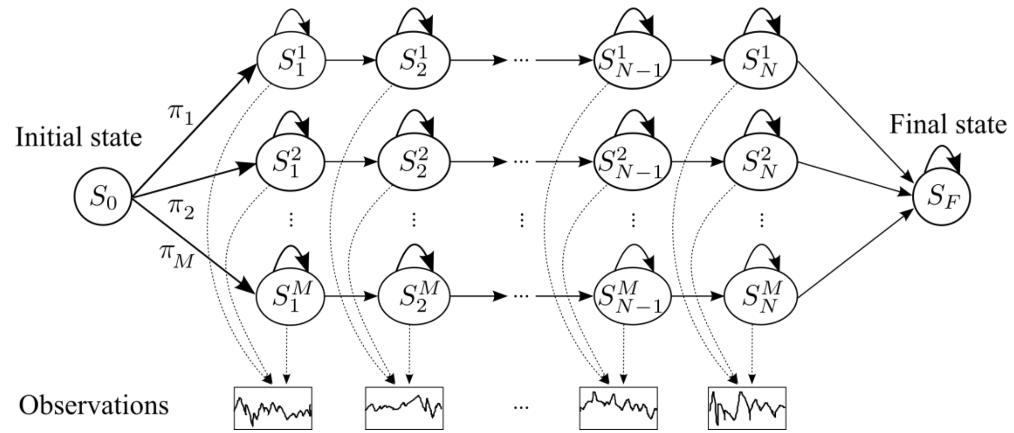
Outline

- Problem statement
- Multi-branch discrete-state models
- Jump Markov linear systems
- Conclusion & Perspectives

Multi-branch discrete-state modeling

Model construction

- M modes => M branches in parallel
- States are unobservable
- Observations : probabilistic links



Multi-branch discrete-state model

Assumptions

- Monotonic deterioration: left-right topology
- Different modes can pass the same health states before the failure
- Exclusive deterioration modes once initiated => No branches switching

Multi-branch discrete-state modeling

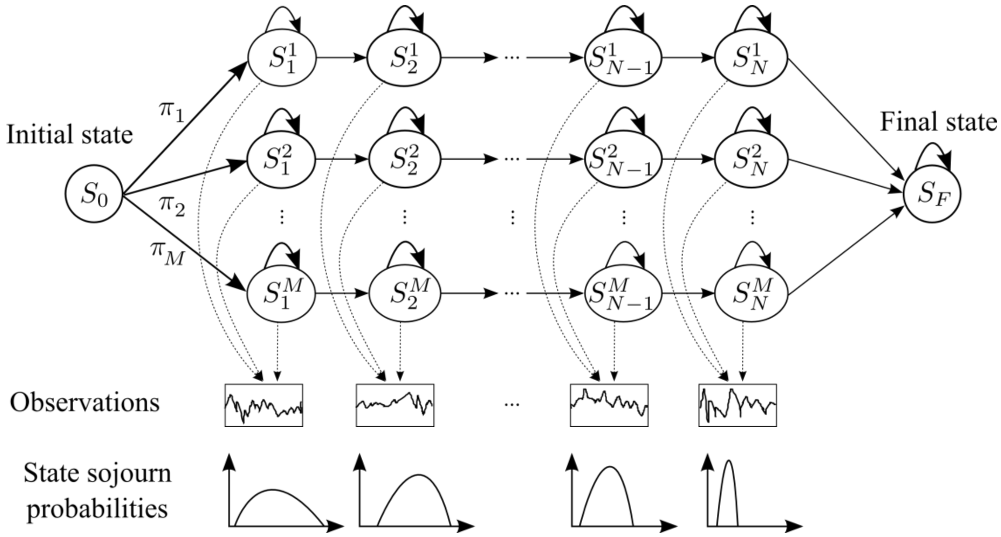
Multi-branch Hidden Markov Model (MB-HMM)

- Each branch ~ left-right HMM
- Markovian property

=> state sojourn time ~ exponential

or geometrical distributions

=> May not be realistic



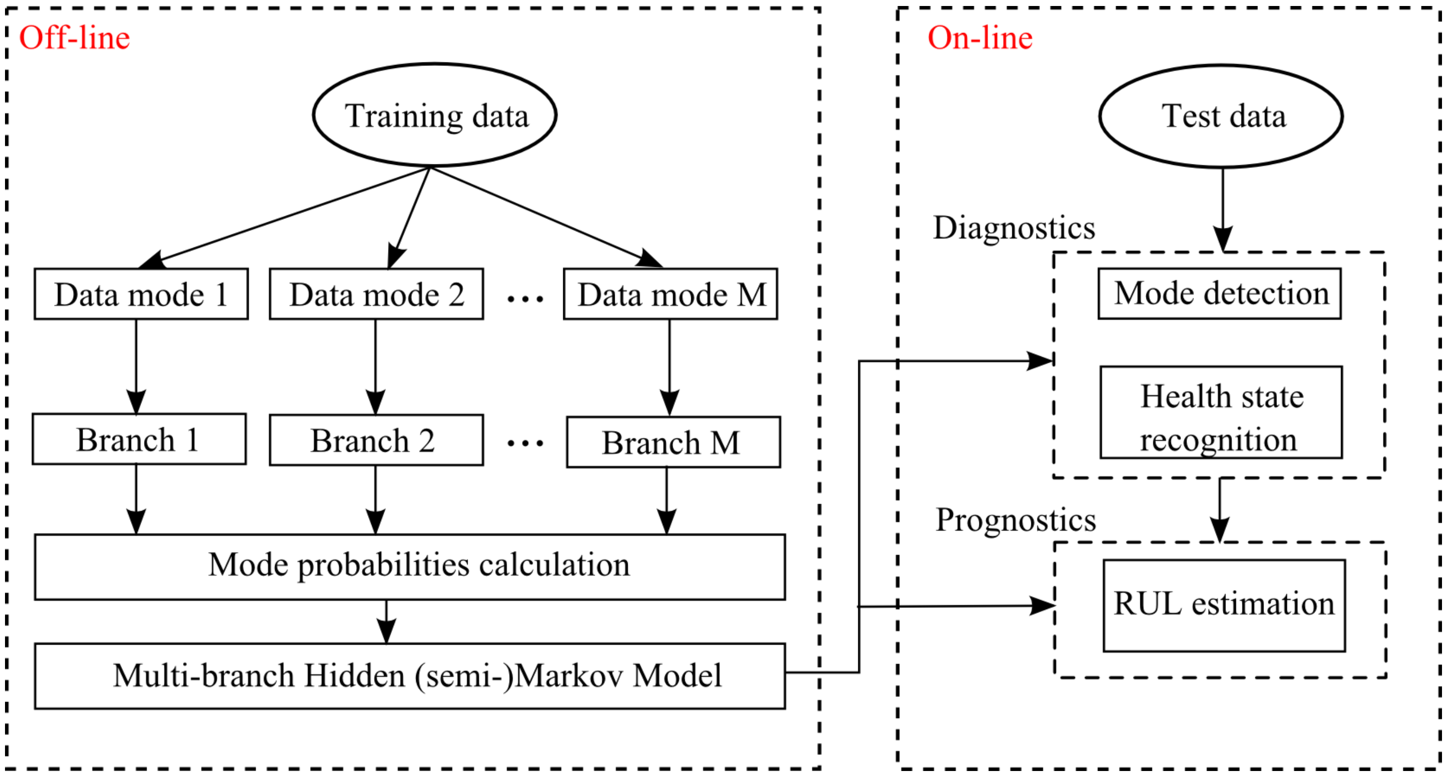
Multi-branch Hidden semi-Markov Model (MB-HsMM)

- Each branch ~ left-right HsMM

=> allow arbitrary distributions for state sojourn time: Gaussian, Weibull, …

Diagnostics and prognostics framework

Two-phase implementation: offline & online



Off-line phase

Model training

- Training data: high-level features
- Data classification => train each branch separately
  - MB-HMM: Baum-Welch algorithm adaption
  - MB-HsMM: Forward-Backward procedure adaption [Yu06]
- *A priori* mode probabilities

$$\pi_k = P(\lambda_k) = K_k / K , \quad k = 1 \ldots M$$

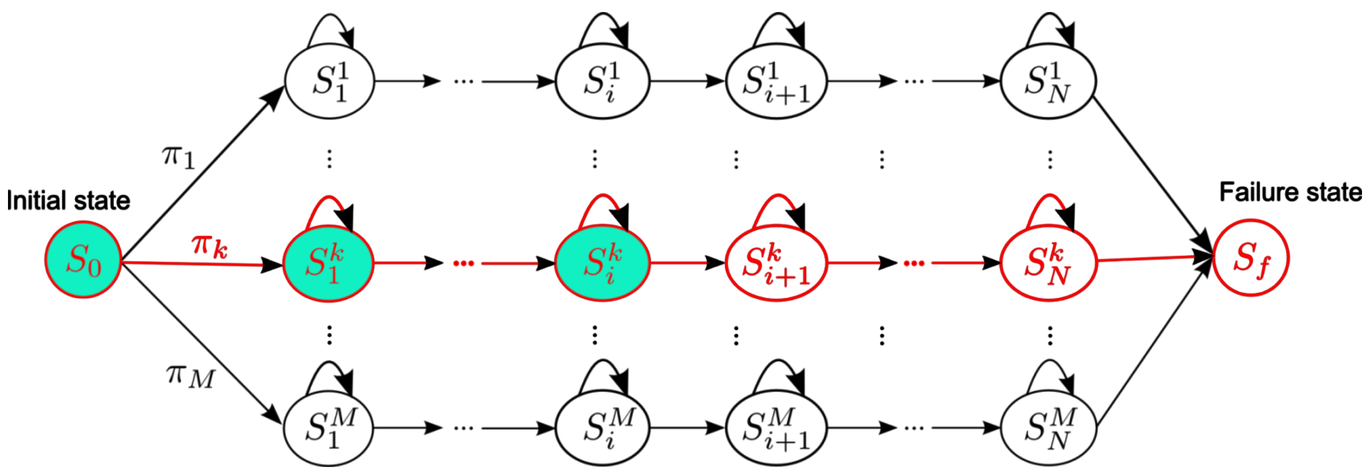
K_k : number of training sequences corresponding to the mode k

*[Yu06]: Practical implementation of an efficient forward-backward algorithm for an explicit-duration hidden Markov model. *IEEE Transactions on Signal Processing*, *54*(5), p. 1947-1951.

On-line phase

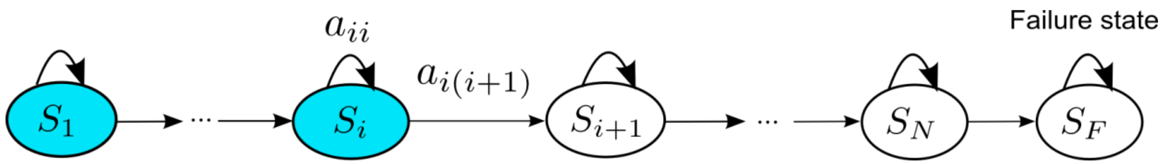
Diagnostics

- Mode detection: $\hat{k} = \arg\max_{k} P\left(\lambda_k \mid \mathbf{O}\right)$
- Health-state assessment: Viterbi algorithm
  - Determine the "best" state sequence: $Q^* = \arg\max_{Q_{\hat{k}}} P\left(\mathbf{O}, Q_{\hat{k}} \mid \lambda_{\hat{k}}\right)$
  - Actual health state = last state in Q^*



RUL estimation

One branch (HMM case)



- Suppose that the system is in state S_i of mode *k*
- Discrete time: RUL = time to hit state S_f ***for the 1st time***

$$RUL_i^{(l)} = P\left(RUL = l \mid q_t = S_i\right) = P\left(q_{t+l} = S_F, q_{t+l-1} \neq S_F, \ldots, q_{t+1} \neq S_F \mid q_t = S_i\right)$$

- Strictly left-right: Given S_i, the system can either stay in S_i or jump to S_{i+1}

$$RUL_i^{(l)} = a_{ii} RUL_i^{(l-1)} + a_{i(i+1)} RUL_{i+1}^{(l-1)}$$

$\Rightarrow$ ***Recursive computation***

RUL estimation *(cont.)*

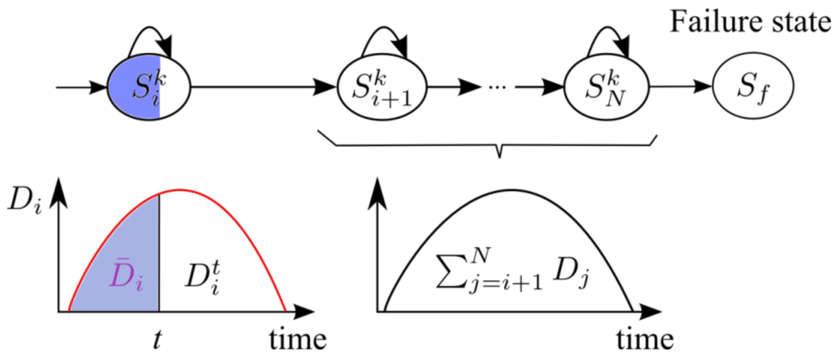
One branch (HsMM case)

- Strictly left-right:

$$\mathrm{RUL}_i^t = D_i^t + \sum_{j=i+1}^{N} D_j$$

D_j : sojourn time in states *j*

$D_i^t = D_i - \bar{D}_i \mid D_i > \bar{D}_i$ ~ truncated distribution



- Gaussian assumption: $\sum_{j=i+1}^{N} D_j$ ~ Normal distribution

RUL estimation *(cont.)*

Multi-branch: bayesian model averaging

- Take into account model uncertainty

$$P(\mathrm{RUL} \mid \mathbf{O}) = \sum_{k=1}^{M} P(\mathrm{RUL} \mid \lambda_k, \mathbf{O}) P(\lambda_k \mid \mathbf{O})$$

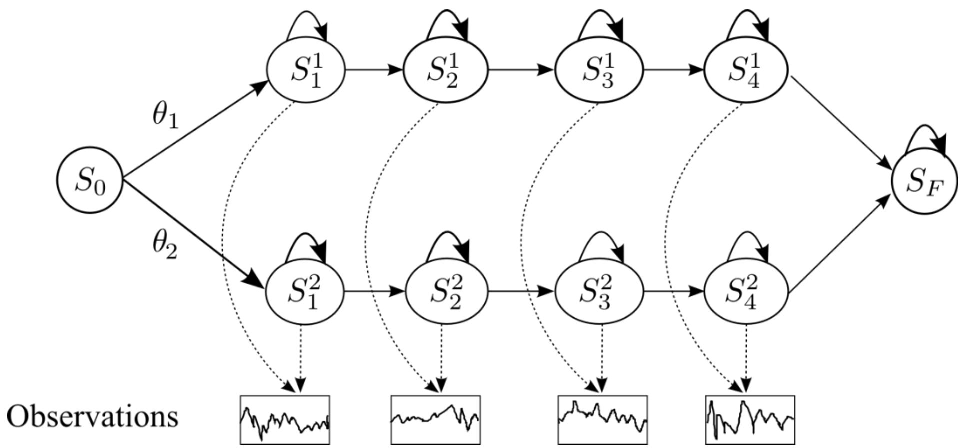
where $P(\lambda_k \mid \mathbf{O}) = \dfrac{P(\mathbf{O} \mid \lambda_k) P(\lambda_k)}{\sum_{k=1}^{M} P(\mathbf{O} \mid \lambda_k) P(\lambda_k)}$: posteriori mode probability

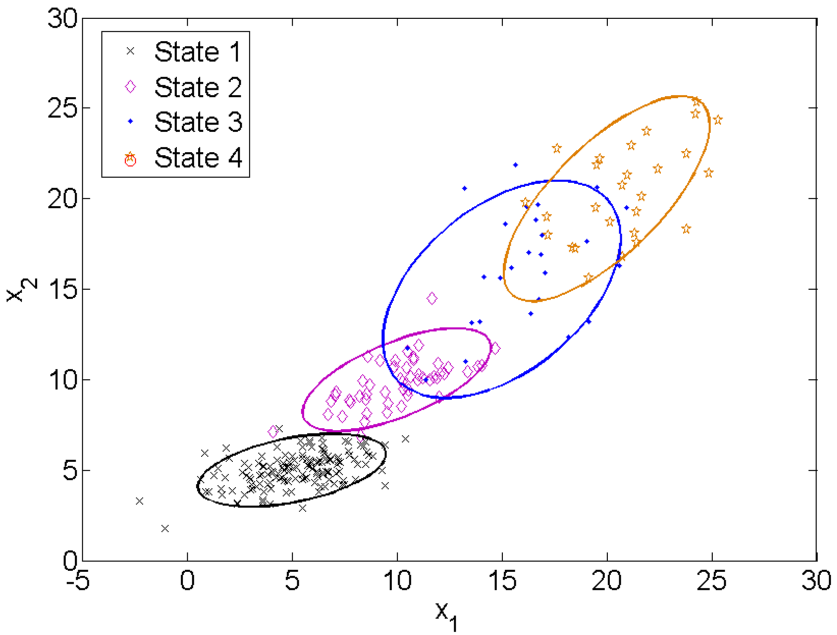
Numerical results

Synthetized data

- 2-branch HMM model
- 4 states for each branch
- Observations are continuous

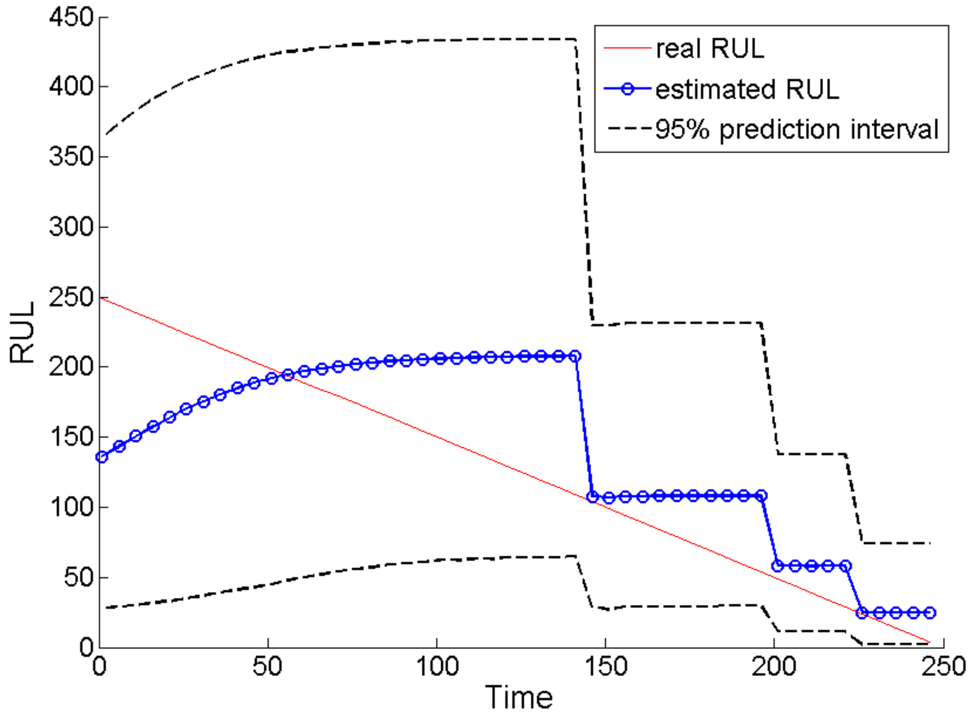
~ bivariate Gaussian distribution



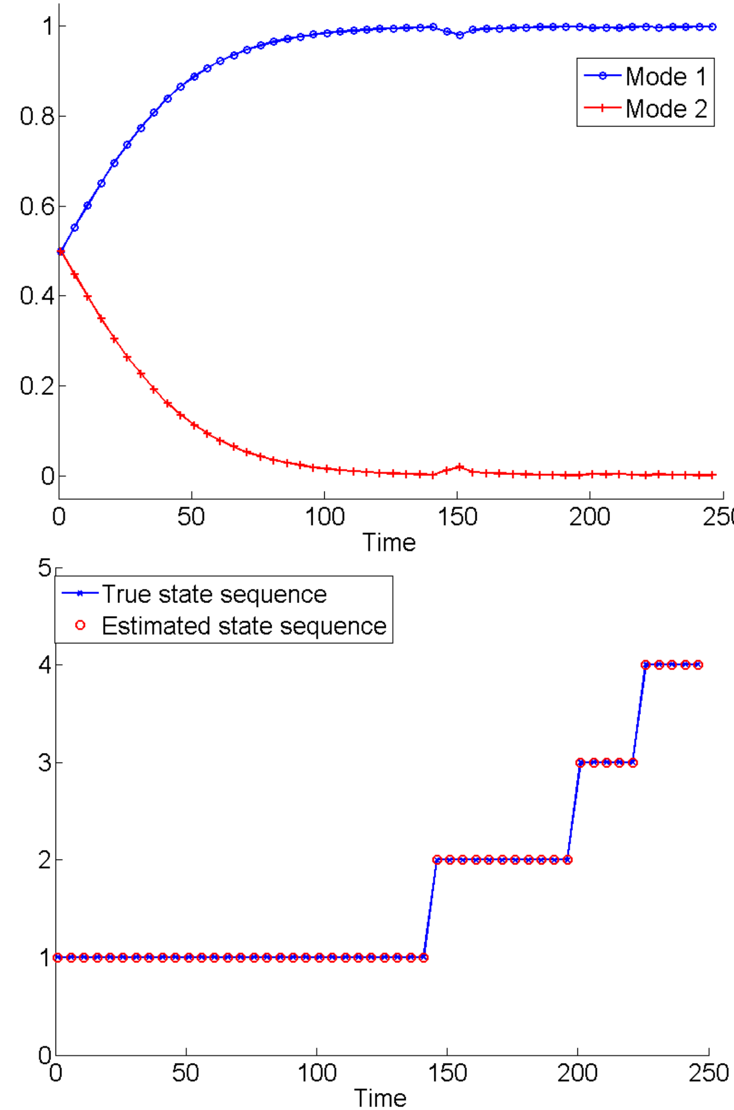


Synthetized deterioration data

Numerical results



RUL estimation



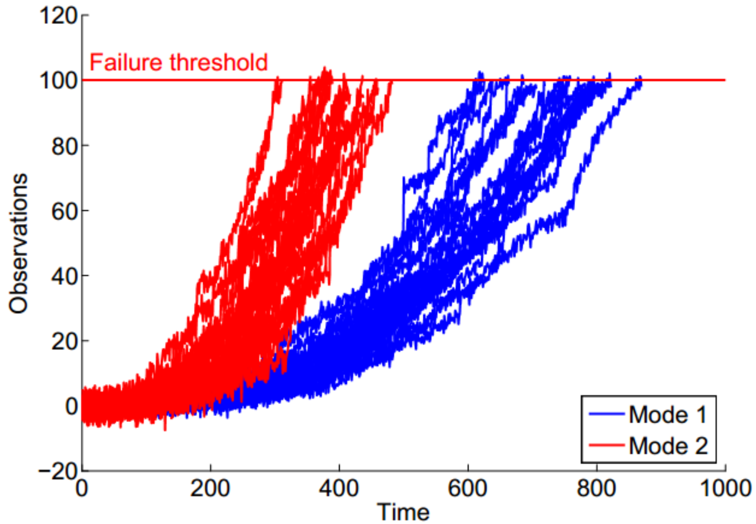
Mode and health state estimation

Numerical results

Fatigued crack evolution

- Continuous => test of robustness
- Fatigue crack growth (FCG) model
  - Stochastic version: $x_{t_i} = x_{t_{i-1}} + e^{w_{t_i}} C\left(\beta\sqrt{x_{t_{i-1}}}\right)^n \Delta t$
  - Observations: $y_{t_i} = x_{t_i} + \xi_{t_i}$
- Multi modes: $\beta(\varepsilon) = \beta_b \cdot e^{\gamma_\varepsilon}$
  - Mode 1 (slow) : $\gamma_1 = 0$
  - Mode 2 (quick) : $\gamma_2 = 0.75$

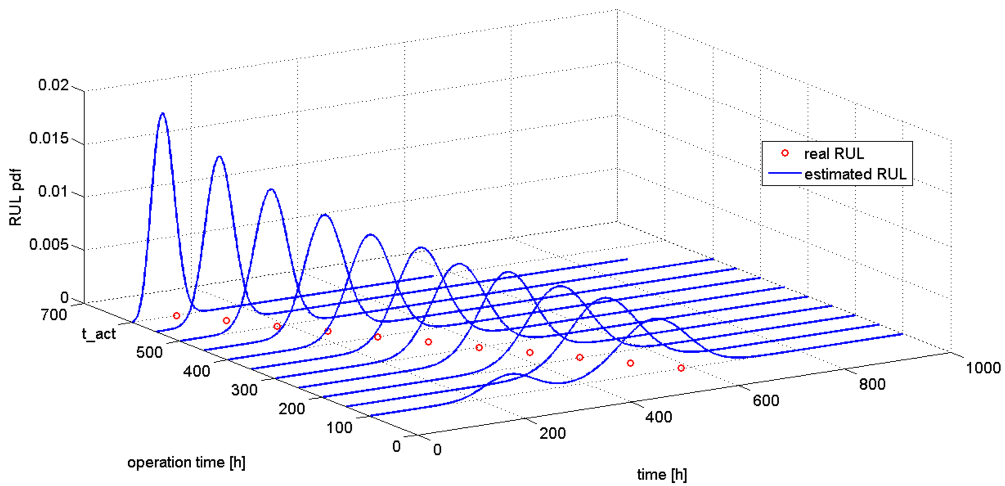
γ_ε : environment factor



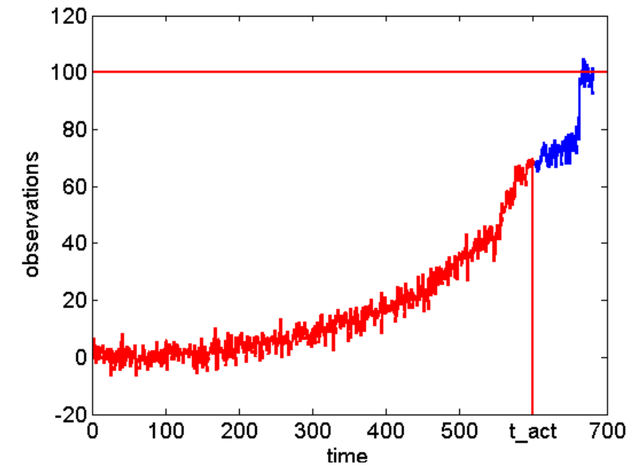
Two-mode deterioration data

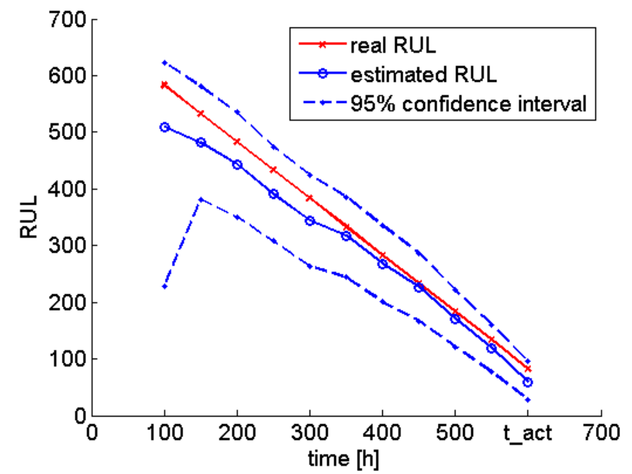
Numerical results

Online RUL estimation (MB-HsMM model)



RUL pdf



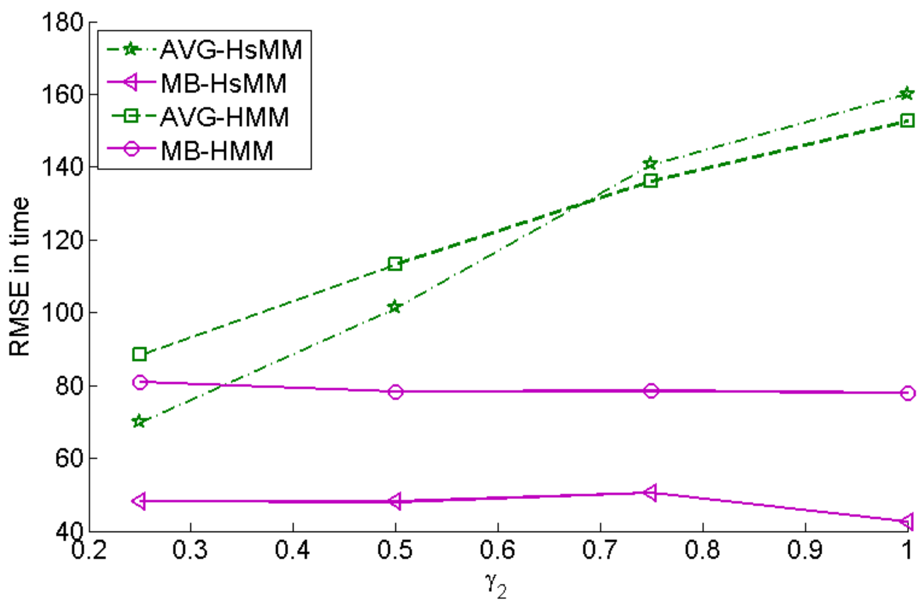


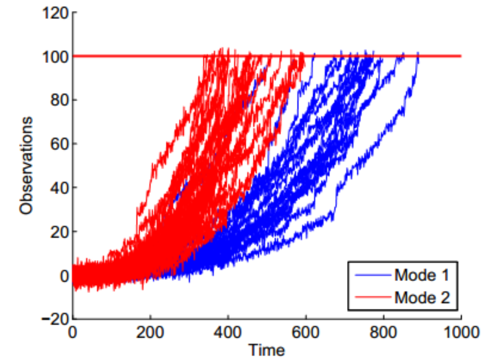
Mean value

Numerical results

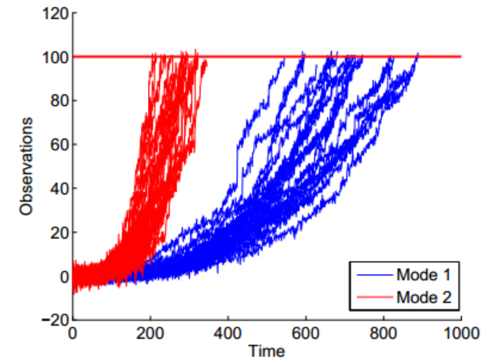
Multi-branch vs. Average model

- Mode “distance”: difference between deterioration rates
- FCG data: fix $\gamma_1 = 0$ => mode distance ~ γ_2
- RUL estimation performance





Small distance $\gamma_2 = 0.4$



Large distance $\gamma_2 = 0.8$

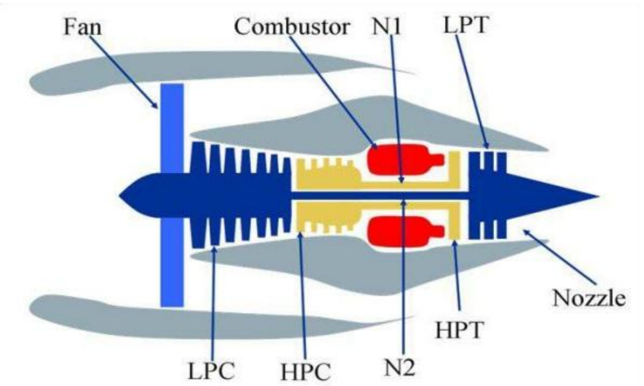
Case study (MB-HsMM model)

PHM08 competition

- C-MAPSS: large realistic commercial turbofan engine
- 2 data set: training & test
- One set: 218 identical and independent units
- Objectives:
  - Construct a prognostic method basing on training data set
  - Use it to estimate the RUL of each unit in test data set
- Evaluation criterion: $S = \sum_{i=1}^{218} S_i$

$$S_i = \begin{cases} e^{-d_i/13} - 1, & d_i \le 0 \\ e^{d_i/10} - 1, & d_i > 0 \end{cases}$$: penalty score

$$d_i = RUL_{est}^i - RUL_{real}^i$$



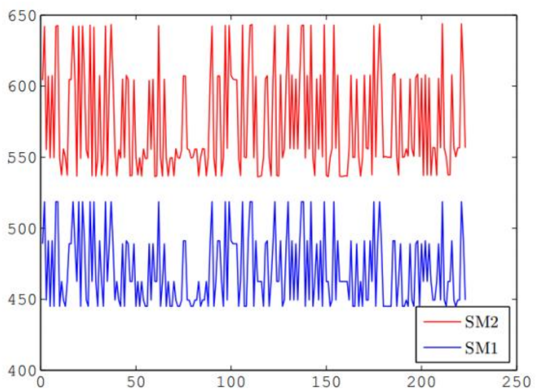
Simplified diagram of engine simulated in C-MAPSS

**C-MAPSS: Commercial Modular Aero-Propulsion System Simulation*

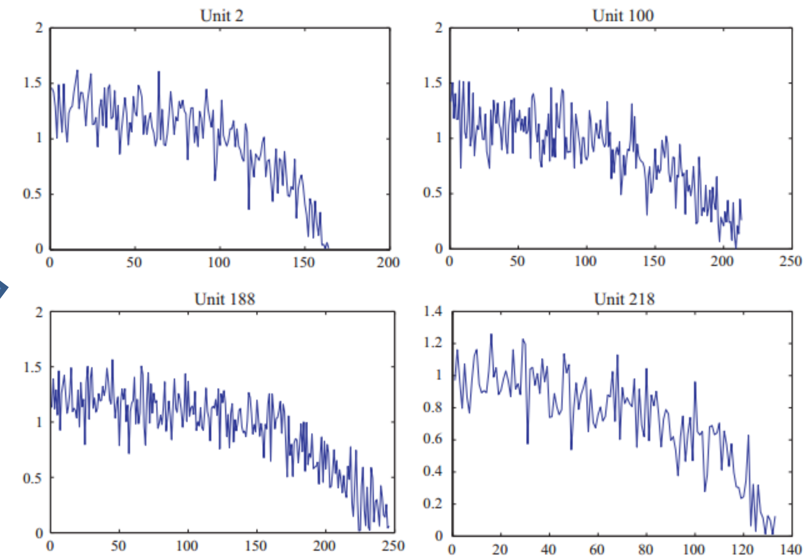
PHM08 data

Health indicator construction

- From [Le Son *et al.*]*



PCA-based



Health indicator evolution

- Clear tendency
- Better score than competition winners

* [Le Son *et al.*] Remaining useful life estimation based on stochastic deterioration models: A comparative study. *Reliability Engineering & System Safety 2012*

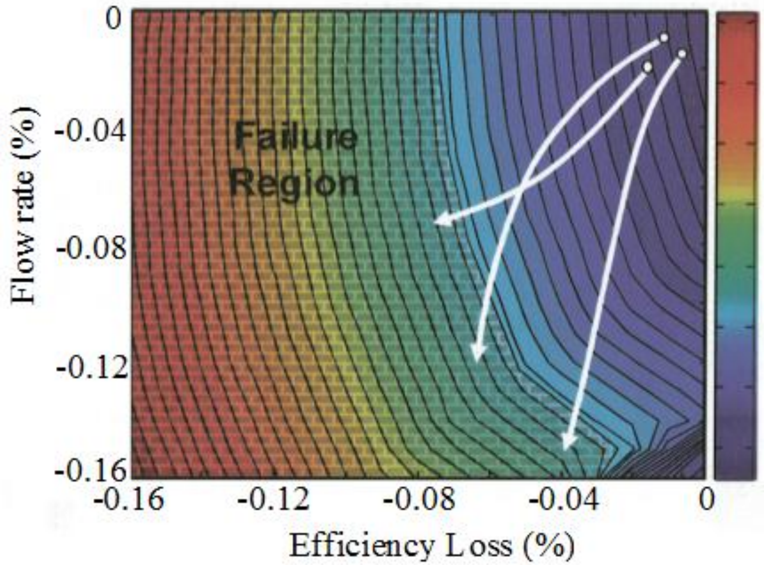
Application of the MB-HsMM model

Number of deterioration modes

- Different fault propagation trajectories depending on the decrease rates of the flow rate (f) and efficiency (e) parameters

- 3 scenarios

	2 modes	3 modes	4 modes
Mode 1	f < e	f < e	f << e
Mode 2	f > e	f ≈ e	f < e
Mode 3		f > e	f > e
Mode 4			f >> e
No branches	2	3	4



Fault propagation trajectories

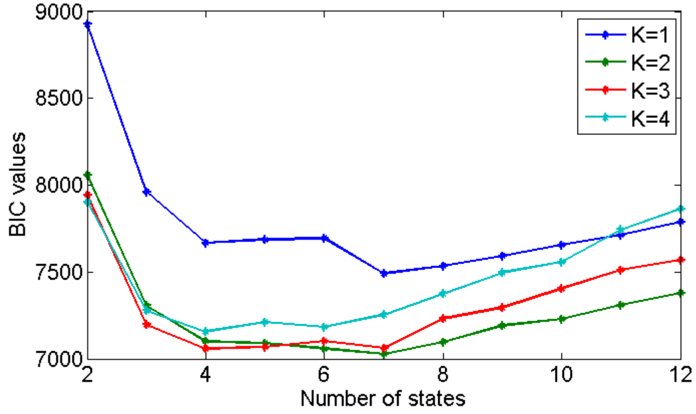
Application of the MB-HsMM model

Topology selection

- Observation model: mixture of Gaussian

$$b_j(\mathbf{x}) = \sum_{k=1}^{K} c_{jk} \mathrm{N}\left(\mathbf{x}; \mu_{jk}, \Sigma_{jk}\right)$$

- Bayesian information criterion (BIC): N = 7; K = 2



RUL estimation result

Method	Score	MSE
1-branch HsMM	12246	1157
2-branch HsMM	6456	936
3-branch HsMM	**5458**	**773**
4-branch HsMM	**3791**	**694**
Wiener-based method	5575	823
Gamma-based method	4107	864

Multi-branch discrete-state modeling

Summary

- Discrete health states => easy to interpret
- Multi-branch: take into account the co-existence of several deterioration modes
- Better RUL estimation performance
- No mode switching once initiated !

Outline

- Problem statement
- Multi-branch discrete-state models
- Jump Markov linear systems
- Conclusion & Perspectives

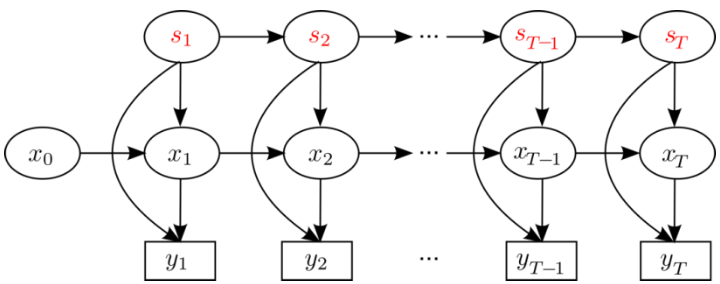
Introduction

- Health states: continuous, unobservable
- Deterioration modes transitions, i.e. load dependent deterioration

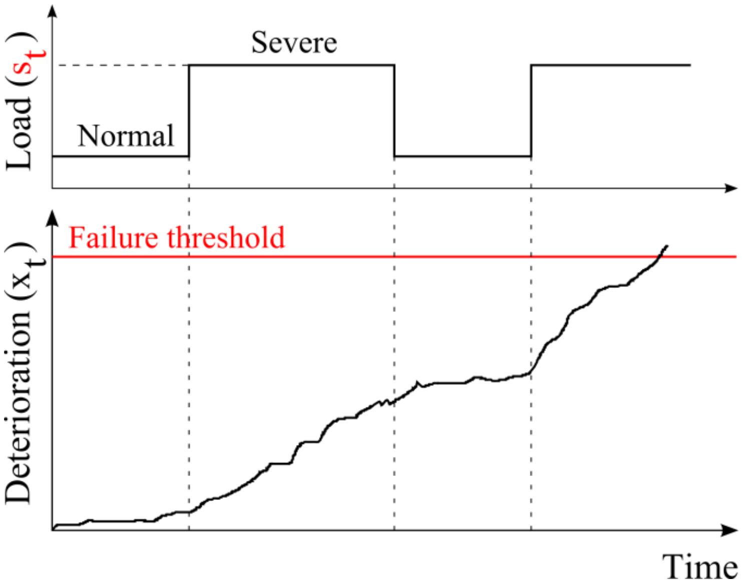
=> Switching state-space model:

$$\begin{cases} x_t = f(x_{t-1}, \omega_t, s_t) \\ y_t = g(x_t, v_t, s_t) \end{cases}$$

S_t : realization at time t of discrete variable **S**



Switching state-space model



Load-dependent deterioration

- General model=> linear approximation

Jump Markov Linear Systems

- Linear switching state-space model:

$$\begin{cases} x_t = A_{s_t} x_{t-1} + \omega_t \\ y_t = C_{s_t} x_t + v_t \end{cases} \quad \text{where} \quad \begin{cases} \omega_t \sim \mathrm{N}\left(0, Q_{s_t}\right) \\ v_t \sim \mathrm{N}\left(0, R_{s_t}\right) \\ x_0 : \mathrm{N}\left(\mu_0, \Sigma_0\right) \end{cases}$$

- M deterioration modes: $s_t \in \{1, 2, \ldots, M\}$
- Mode transitions ~ discrete-time Markov chain
  - Transition matrix:

$$\Pi = \begin{pmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1M} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{M1} & \pi_{M2} & \cdots & \pi_{MM} \end{pmatrix} \quad \text{where} \quad \pi_{ij} = P\left(s_{t+1} = j \mid s_t = i\right)$$

  - Initial distribution: $\pi_1(i) = P(s_1 = i)$
- Identifiability guarantee: C_{s_t} fixed

Parameters learning problem

Model parameters

$$\Theta = \left\{ \left(A_i, Q_i, R_i \right)_{i=1,\ldots,M}, \mu_0, \Sigma_0, \Pi, \pi_1 \right\}$$

- X, S are hidden => Expectation-Maximization algorithm
  - E step: $\mathrm{Q}\left(\Theta \mid \Theta^{(k)}\right) = \mathbf{E}\left[\log \mathrm{P}\left(\mathrm{X}_T, \mathrm{S}_T, \mathrm{Y}_T \mid \Theta\right) \mid \mathrm{Y}_T, \Theta^{(k)} \right]$
  - M step: $\Theta^{(k+1)} = \arg\max_{\Theta} \mathrm{Q}\left(\Theta \mid \Theta^{(k)}\right)$
- Problem: presence of switching dynamic

$$\mathrm{Q}\left(\Theta \mid \Theta^{(k)}\right) = \sum_{\mathrm{S}_T} \left(\mathrm{P}\left(\mathrm{S}_T \mid \mathrm{Y}_T, \Theta^{(k)}\right) \int p\left(\mathrm{X}_T \mid \mathrm{S}_T, \mathrm{Y}_T, \Theta^{(k)}\right) \log \mathrm{P}\left(\mathrm{X}_T, \mathrm{S}_T, \mathrm{Y}_T \mid \Theta\right) d\mathrm{X}_T \right)$$

=> Computed over all possible sequences of discrete states S_T => Intractable

=> Approximated EM algorithm

Approximated EM algorithm

Pruning technique

- Idea: Calculate Q over the most “likely” state sequence

=> Adaption of the Viterbi algorithm

- Do not guarantee the convergence, but still sufficient in several practical cases

Approximated Q function

$$Q\left(\Theta | \Theta^{(k)}\right) \approx \int p\left(X_T | S_T^*, Y_T, \Theta^{(k)}\right) \log P\left(X_T, S_T^*, Y_T | \Theta\right) dX_T$$

S_T^* : the most likely state sequence

=> Calculated by Rauch-Tung-Streiber (RTS) smoother

JMLS based diagnostics

- Mode probabilities

$$\mu_t(i) = P\left(s_t = i\right) = \frac{P\left(\mathrm{S}_{t,i}^*\right)}{\sum_{i=1}^{M} P\left(\mathrm{S}_{t,i}^*\right)}$$

$\mathrm{S}_{t,i}^*$: the best state sequence at time t that ends in state i

- Health state assessment: mixture of mode-dependent estimated states

$$\begin{cases} \hat{x}_t = \sum_{i=1}^{M} \mu_t(i)\hat{x}_{t,i} \\ \hat{\Sigma}_t = \sum_{i=1}^{M} \mu_t(i)\hat{\Sigma}_{t,i} + \sum_{i=1}^{M} \mu_t(i)\left(\hat{x}_{t,i} - \hat{x}_t\right)\left(\hat{x}_{t,i} - \hat{x}_t\right)^T \end{cases}$$

RUL prediction

- Discrete time: $RUL_t = \left(\min k : x_{t+k} \geq L \mid x_t < L\right)$
- Mode switching: M fold increase in number of Gaussian distributions

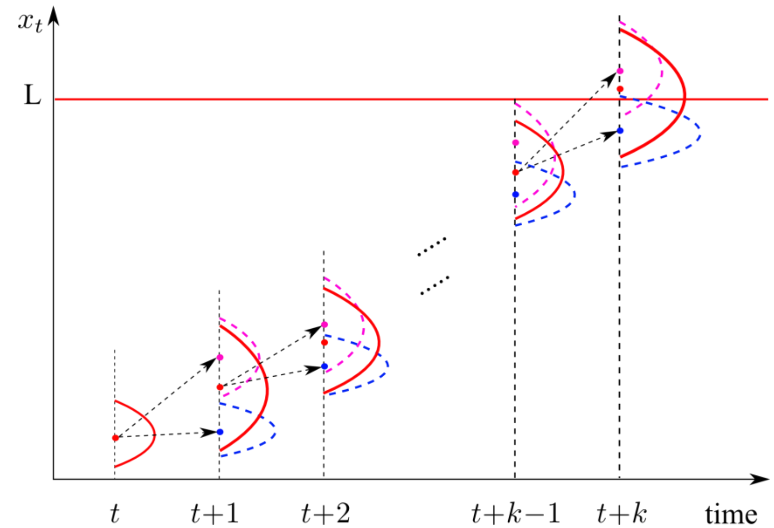
$\Rightarrow$ Intractable computation

$\Rightarrow$ Approximation: merge all one-step predicted Gaussian distributions into one

$$p\left(x_{t+1|t}\right) \approx \sum_{i=1}^{M} \mu_{t+1}(i) \mathrm{N}\left(x_{t+1|t,i}, \Sigma_{t+1|t,i}\right)$$

- Mode probability update

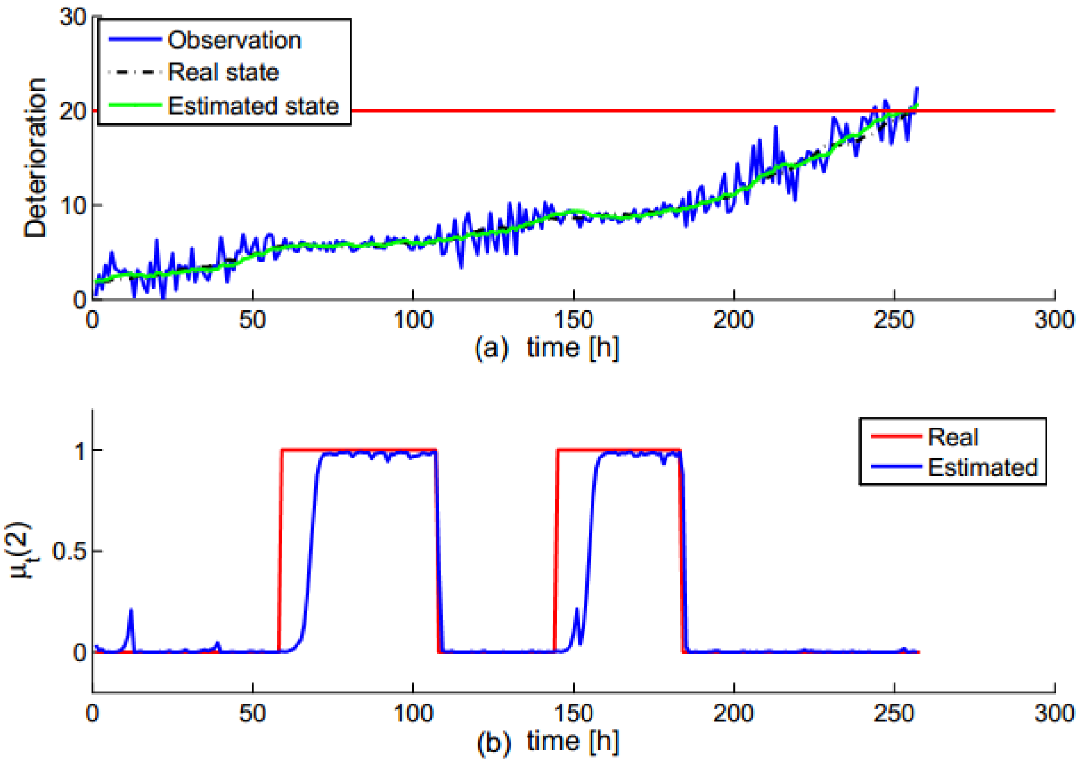
$$\mu_{t+1}(i) = \sum_{j=1}^{M} \pi_{j,i} \cdot \mu_t(j)$$



RUL prediction illustration for 2-mode case

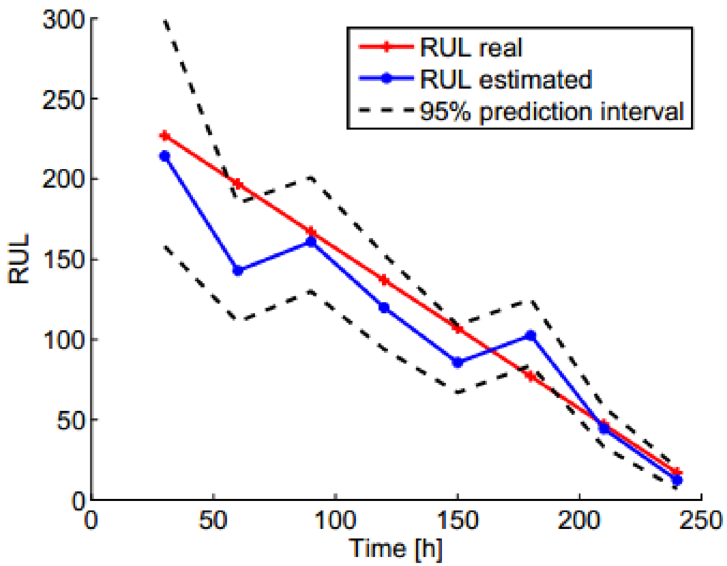
Diagnostics example

Mode detection and health assessment

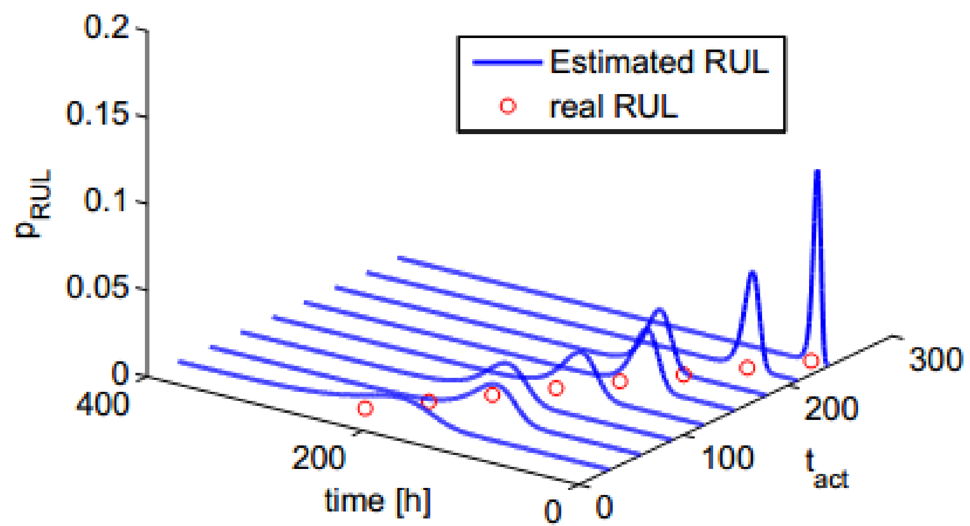


RUL estimation example

Online prediction



Mean estimated RUL at different times



Evolution of estimated RUL pdf

Outline

- Problem statement
- Multi-branch discrete-state models
- Jump Markov linear systems
- Conclusion & Perspectives

Conclusion & Perspectives

Conclusion

- Co-existence of multiple deterioration modes => multi-branch modeling
  - Discrete states: MB-HMM & MB-HsMM models
  - Continuous states: JMLS model
- Development of corresponding diagnostics & prognostics framework
  - Detection of the actual deterioration mode
  - Assessment of the current health status
  - Estimation of the RUL

Conclusion & Perspectives

Perspectives

- MB-HMM & MB-HsMM models
  - Relax the left-right assumption
  - Allow mode switching
- JMLS model
  - Extension to non-linear models
  - Improvement of model learning methods
  - Semi-Markov mode transitions
- Dynamic adaptation of maintenance strategies based on RUL estimation results
- Further model validation on real-life data

Thank you for your attention!